12th STD.

MARCH - 2017

[Time : 3 hours] **Mathematics** (With Answers) [Marks : 200]

PART - A

Note: (i) **All** questions are **compulsory.**

(ii) Choose the most suitable answer from the given **four** alternatives and write the option code and the corresponding answer. **[40 × 1 = 40]**

1. The differential equation $\left(\frac{dx}{dy}\right)^2 + 5y^{\frac{1}{3}} = x$ is :
   (a) of order 2 and degree 1
   (b) of order 1 and degree 2
   (c) of order 1 and degree 6
   (d) of order 1 and degree 3

2. If $(m-5) + i(n+4)$ is the complex conjugate of $(2m+3) + i(3n-2)$ then (n, m) are :
   (a) $\left(\frac{-1}{2}, -8\right)$ (b) $\left(\frac{-1}{2}, 8\right)$
   (c) $\left(\frac{1}{2}, -8\right)$ (d) $\left(\frac{1}{2}, 8\right)$

3. The principal value of arg (z) lies in the interval:
   (a) $\left[0, \frac{\pi}{2}\right]$ (b) $(-\pi, \pi]$
   (c) $[0, \pi]$ (d) $(-\pi, 0]$

4. Volume of the solid obtained by revolving the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about major and minor axes are in the ratio :
   (a) $b^2:a^2$ (b) $a^2:b^2$ (c) $a:b$ (d) $b:a$

5. The curve $y^2(x-2) = x^2(1+x)$ has :
   (a) an asymptote parallel to x-axis
   (b) an asymptote parallel to y-axis
   (c) asymptotes parallel to both axes
   (d) no asymptote

6. In the multiplicative group of nth roots of unity, the inverse of ω^k is $(k < n)$:
   (a) $\omega^{\frac{1}{k}}$ (b) ω^{-1} (c) ω^{n-k} (d) $\omega^{\frac{n}{k}}$

7. The equation of the line parallel to $\frac{x-3}{1} = \frac{y+3}{5} = \frac{2z-5}{3}$ and passing through the point (1, 3, 5) in vector form, is :
   (a) $\vec{r} = (\vec{i} + 5\vec{j} + 3\vec{k}) + t(\vec{i} + 3\vec{j} + 5\vec{k})$
   (b) $\vec{r} = (\vec{i} + 3\vec{j} + 5\vec{k}) + t(\vec{i} + 5\vec{j} + 3\vec{k})$
   (c) $\vec{r} = \left(\vec{i} + 5\vec{j} + \frac{3}{2}\vec{k}\right) + t(\vec{i} + 3\vec{j} + 5\vec{k})$
   (d) $\vec{r} = (\vec{i} + 3\vec{j} + 5\vec{k}) + t\left(\vec{i} + 5\vec{j} + \frac{3}{2}\vec{k}\right)$

8. The distance - time relationship of a moving body is given by $y = F(t)$ then the acceleration of the body is the :
   (a) Gradient of the velocity / time graph
   (b) Gradient of the distance / time graph
   (c) Gradient of the acceleration / time graph
   (d) Gradient of the velocity/ distance graph

9. If $f(x) = x^2 - 4x + 5$ on [0, 3] then the absolute maximum value is :
   (a) 2 (b) 3 (c) 4 (d) 5

10. If p′s truth value is T and q′s truth value is F, then which of the following have the truth value T ?
    (i) $p \vee q$ (ii) $\sim p \vee q$ (iii) $p \vee (\sim q)$ (iv) $p \wedge (\sim q)$
    (a) (i), (ii), (iii) (b) (i), (ii), (iv)
    (c) (i), (iii), (iv) (d) (ii), (iii), (iv)

11. One of the foci of the rectangular hyperbola $xy = 18$ is :
    (a) (6, 6) (b) (3, 3) (c) (4, 4) (d) (5, 5)

12. In echelon form, which of the following is incorrect ?
    (a) Every row of A which has all its entries 0 occurs below every row which has a non-zero entry.
    (b) The first non-zero entry in each non-zero row is 1.
    (c) The number of zeros before the first non-zero element in a row is less than the number of such zeros in the next row.
    (d) Two rows can have same number of zeros before the first non-zero entry

13. Solution of $\frac{dx}{dy} + mx = 0$, where $m < 0$ is :

(a) $x = ce^{my}$ (b) $x = ce^{-my}$

(c) $x = my + c$ (d) $x = c$

14. The random variable X follows normal distribution $f(x) = ce^{-\frac{1}{2}\frac{(x-100)^2}{25}}$. Then the value of c is :

(a) $\sqrt{2\pi}$ (b) $\frac{1}{\sqrt{2\pi}}$ (c) $5\sqrt{2\pi}$ (d) $\frac{1}{5\sqrt{2\pi}}$

15. If a line makes 45°, 60° with positive direction of axes x and y then the angle it makes with the z-axis is :

(a) 30° (b) 90° (c) 45° (d) 60°

16. The value of $[\vec{i}+\vec{j}, \vec{j}+\vec{k}, \vec{k}+\vec{i}]$ is equal to :

(a) 0 (b) 1 (c) 2 (d) 4

17. The value of $\int_0^{\frac{\pi}{4}} \cos^3 2x\, dx$ is :

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) 0 (d) $\frac{2\pi}{3}$

18. A random variable X has the following probability distribution :

X	0	1	2	3	4	5
P(X = x)	$\frac{1}{4}$	$2a$	$3a$	$4a$	$5a$	$\frac{1}{4}$

Then $P(1 \le X \le 4)$ is :

(a) $\frac{10}{21}$ (b) $\frac{2}{7}$ (c) $\frac{1}{14}$ (d) $\frac{1}{2}$

19. The mean of a binomial distribution is 5 and its standard deviation is 2. Then the values of n and p are :

(a) $\left(\frac{4}{5}, 25\right)$ (b) $\left(25, \frac{4}{5}\right)$

(c) $\left(\frac{1}{5}, 25\right)$ (d) $\left(25, \frac{1}{5}\right)$

20. If x_0 is the x-coordinate of the point of inflection of curve $y = f(x)$ then (assume second derivative exists) :

(a) $f(x_0) = 0$ (b) $f'(x_0) = 0$

(c) $f''(x_0) = 0$ (d) $f''(x_0) \neq 0$

21. The value of 'c' in Rolle's Theorem for the function $f(x) = \cos\frac{x}{2}$ on $[\pi, 3\pi]$ is :

(a) 0 (b) 2π (c) $\frac{\pi}{2}$ (d) $\frac{3\pi}{2}$

22. The normal at 't_1' on the parabola $y^2 = 4ax$ meets the parabola at 't_2' then $\left(t_1 + \frac{2}{t_1}\right)$ is :

(a) $-t_2$ (b) t_2 (c) $t_1 + t_2$ (d) $\frac{1}{t_2}$

23. If ω is the cube root of unity then the value of $(1-\omega)(1-\omega^2)(1-\omega^4)(1-\omega^8)$ is :

(a) 9 (b) –9 (c) 16 (d) 32

24. If $\overrightarrow{PR} = 2\vec{i}+\vec{j}+\vec{k}$, $\overrightarrow{QS} = -\vec{i}+3\vec{j}+2\vec{k}$ then the area of the quadrilateral PQRS is :

(a) $5\sqrt{3}$ (b) $10\sqrt{3}$ (c) $\frac{5\sqrt{3}}{2}$ (d) $\frac{3}{2}$

25. The eccentricity of the conic $9x^2+5y^2-54x-40y+116=0$ is :

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{4}{9}$ (d) $\frac{2}{\sqrt{5}}$

26. The order of [7] in $(z_9, +_9)$ is :

(a) 9 (b) 6 (c) 3 (d) 1

27. If P represents the variable complex number z and if $|2z - 1| = 2|z|$ then the locus of P is :

(a) the straight line $x = \frac{1}{4}$

(b) the straight line $y = \frac{1}{4}$

(c) the straight line $z = \frac{1}{2}$

(d) the circle $x^2 + y^2 - 4x - 1 = 0$

28. A continuous random variable X has p.d.f. $f(x)$, then :
(a) $0 \le f(x) \le 1$ (b) $f(x) \ge 0$
(c) $f(x) \le 1$ (d) $0 < f(x) < 1$

29. The asymptotes of the hyperbola $36y^2 - 25x^2 + 900 = 0$, are:
(a) $y = \pm\frac{6}{5}x$ (b) $y = \pm\frac{5}{6}x$
(c) $y = \pm\frac{36}{25}x$ (d) $y = \pm\frac{25}{36}x$

30. If $I_n = \int \cos^n x\, dx$ then $I_n =$
(a) $\frac{-1}{n}\cos^{n-1}x\sin x + \left(\frac{n-1}{n}\right)I_{n-2}$
(b) $\cos^{n-1}x\sin x + \left(\frac{n-1}{n}\right)I_{n-2}$
(c) $\frac{1}{n}\cos^{n-1}x\sin x - \left(\frac{n-1}{n}\right)I_{n-2}$
(d) $\frac{1}{n}\cos^{n-1}x\sin x + \left(\frac{n-1}{n}\right)I_{n-2}$

31. The value of $\int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1+\sin x\cos x}dx$ is :
(a) $\frac{\pi}{2}$ (b) 0 (c) $\frac{\pi}{4}$ (d) π

32. If $y = ke^{\lambda x}$ then its differential equation is (where k is arbitrary constant) :
(a) $\frac{dy}{dx} = \lambda y$ (b) $\frac{dy}{dx} = ky$
(c) $\frac{dy}{dx} + ky = 0$ (d) $\frac{dy}{dx} = e^{\lambda x}$

33. If $u = f(x, y)$ is a differentiable function of x and y; where x and y are differentiable functions of 't' then :
(a) $\frac{du}{dt} = \frac{\partial f}{\partial x}.\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}.\frac{\partial y}{\partial t}$
(b) $\frac{du}{dt} = \frac{\partial f}{\partial x}.\frac{dx}{dt} + \frac{\partial f}{\partial y}.\frac{\partial y}{\partial t}$
(c) $\frac{du}{dt} = \frac{\partial f}{\partial x}.\frac{dx}{dt} + \frac{\partial f}{\partial y}.\frac{dy}{dt}$
(d) $\frac{\partial u}{\partial t} = \frac{\partial f}{\partial x}.\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}.\frac{\partial y}{\partial t}$

34. If $A = \begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix}$, then A^{12} is :
(a) $\begin{bmatrix} 0 & 0 \\ 0 & 60 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 0 \\ 0 & 5^{12} \end{bmatrix}$
(c) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

35. The non-parametric vector equation of a plane passing through a point whose position vector is $\vec{a}$ and parallel to $\vec{u}$ and $\vec{v}$, is:
(a) $[\vec{r} - \vec{a}, \vec{u}, \vec{v}] = 0$ (b) $[\vec{r}, \vec{u}, \vec{v}] = 0$
(c) $[\vec{r}, \vec{a}, \vec{u} \times \vec{v}] = 0$ (d) $[\vec{a}, \vec{u}, \vec{v}] = 0$

36. The integrating factor of the differential equation $\frac{dy}{dx} - y\tan x = \cos x$ is:
(a) $\sec x$ (b) $\cos x$ (c) $e^{\tan x}$ (d) $\cot x$

37. If A is a matrix of order 3, then det (kA) is :
(a) k^3 det (A) (b) k^2 det (A)
(c) k det (A) (d) det (A)

38. The point of intersection of the lines $\frac{x-6}{-6} = \frac{y+4}{4} = \frac{z-4}{-8}$ and $\frac{x+1}{2} = \frac{y+2}{4} = \frac{z+3}{-2}$ is :
(a) (0, 0, –4) (b) (1, 0, 0)
(c) (0, 2, 0) (d) (1, 2, 0)

39. If $ae^x + be^y = c$; $pe^x + qe^y = d$ and $\Delta_1 = \begin{vmatrix} a & b \\ p & q \end{vmatrix}$; $\Delta_2 = \begin{vmatrix} c & b \\ d & q \end{vmatrix}$; $\Delta_3 = \begin{vmatrix} a & c \\ p & d \end{vmatrix}$ then the value of (x, y) is :
(a) $\left(\frac{\Delta_2}{\Delta_1}, \frac{\Delta_3}{\Delta_1}\right)$ (b) $\left(\log\frac{\Delta_2}{\Delta_1}, \log\frac{\Delta_3}{\Delta_1}\right)$
(c) $\left(\log\frac{\Delta_1}{\Delta_3}, \log\frac{\Delta_1}{\Delta_2}\right)$ (d) $\left(\log\frac{\Delta_1}{\Delta_2}, \log\frac{\Delta_1}{\Delta_3}\right)$

40. In congruence modulo 5, $\{x \in Z / x = 5k + 2, k \in Z\}$ represents:
(a) [0] (b) [5] (c) [7] (d) [2]

PART - B

Note : (i) Answer **any ten** questions. [10 × 6 = 60]
(ii) Question No. **55** is **compulsory** and choose **any nine** from the remaining.

41. Find the rank of the matrix $\begin{bmatrix} 0 & 1 & 2 & 1 \\ 2 & -3 & 0 & -1 \\ 1 & 1 & -1 & 0 \end{bmatrix}$.

42. Find the inverse of the matrix $\begin{bmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$.

43. Find the point of intersection of the line passing through the two points (1, 1, –1); (–1, 0, 1) and the xy-plane.

44. (i) If $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$, show that $\vec{a}-\vec{d}$ and $\vec{b}-\vec{c}$ are parallel.
(ii) Find the direction cosines of the line joining (2, –3, 1) and (3, 1, –2).

45. If α and β are complex conjugates to each other and $\alpha = -\sqrt{2}+i$ then find $\alpha^2+\beta^2-\alpha\beta$.

46. Show that the points representing the complex numbers $7+9i$, $-3+7i$, $3+3i$ form a right angled triangle on the Argand diagram.

47. A particle of unit mass moves so that displacement after 't' seconds is given by $x = 3\cos(2t-4)$. Find the acceleration and kinetic energy at the end of 2 seconds. [K.E. = $\frac{1}{2}mv^2$, m is mass]

48. (i) Find the critical numbers of $x^{\frac{3}{5}}(4-x)$.
(ii) Determine the domain of convexity of $y = e^x$.

49. The radius of a circular disc is given as 24 cm. with a maximum error in measurement of 0.02 cm. Estimate the maximum error in the calculated area of the disc and compute the relative error by using differentials.

50. Solve : $(D^2 - 4D + 1)y = x^2$

51. Verify whether the statement $q \vee [p \vee (\sim q)]$ is a tautology or a contradiction.

52. Construct the truth table for $(p \wedge q) \vee (\sim r)$.

53. (i) Let Z be a standard normal variate. Find the value of c if $P(Z < c) = 0.05$.
Here $P[0 < Z < 1.65] = 0.45$
(ii) The difference between the mean and the variance of a Binomial distribution is 1 and the difference between their squares is 11. Find n.

54. A die is tossed twice. A success is getting an odd number on a toss. Find the mean and the variance of the probability distribution of the number of successes.

55. (a) Find the equation of the hyperbola if the centre is (2, 5); the distance between the directrices is 15; the distance between the foci is 20 and the transverse axis is parallel to y-axis.

(OR)

(b) Find the volume of the solid obtained by revolving the loop of the curve $2ay^2 = x(x-a)^2$ about x-axis. Here $a > 0$.

PART - C

Note: (*i*) Answer **any ten** questions.**[10 × 10 = 100]**
(*ii*) Question No. **70** is **compulsory** and choose **any nine** from the remaining.

56. Solve $x+y+2z=4$, $2x+2y+4z=8$, $3x+3y+6z=12$ by using determinant method.

57. cos (A + B) = cosA cosB – sinA sinB: prove by vector method.

58. Find the vector and Cartesian equations of the plane passing through the points with position vectors $3\vec{i}+4\vec{j}+2\vec{k}$, $2\vec{i}-2\vec{j}-\vec{k}$ and $7\vec{i}+\vec{k}$.

59. Solve : $x^4 - x^3 + x^2 - x + 1 = 0$

60. On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4 mts when it is 6 mts away from the point of projection. Finally it reaches the ground 12 mts away from the starting point. Find the angle of projection.

61. The ceiling in a hallway 20 ft wide is in the shape of a semi ellipse and 18 ft high at the centre. Find the height of the ceiling 4 feet from either wall if the height of the side walls is 12 ft.

62. Find the equations of the rectangular hyperbola which has for one of its asymptoes the line $x+2y-5=0$ and passes through the points (6, 0) and (–3, 0).

63. Find the point on the parabola $y^2 = 2x$ that is closest to the point (1, 4).

64. If $u=\frac{x}{y^2}-\frac{y}{x^2}$ then verify that $\frac{\partial^2 u}{\partial x\,\partial y}=\frac{\partial^2 u}{\partial y\,\partial x}$.

65. Find the area of the region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, by integration.

66. Find the length of the curve $x = a(t - \sin t)$, $y = a(1 - \cos t)$ between $t = 0$ and $t = \pi$.

67. A cup of coffee at temperature 100°C is placed in a room whose temperature is 15°C and it cools to 60°C in 5 minutes. Find its temperature after a further interval of 5 minutes.

68. Show that

$$\left\{\begin{pmatrix}1&0\\0&1\end{pmatrix},\begin{pmatrix}\omega&0\\0&\omega^2\end{pmatrix},\begin{pmatrix}\omega^2&0\\0&\omega\end{pmatrix},\begin{pmatrix}0&1\\1&0\end{pmatrix},\begin{pmatrix}0&\omega^2\\\omega&0\end{pmatrix},\begin{pmatrix}0&\omega\\\omega^2&0\end{pmatrix}\right\},$$

where $\omega^3=1$, $\omega\neq1$ form a group with respect to matrix multiplication.

69. Verify $f(x)=\begin{cases}30x^4e^{-6x^5}; & x>0\\ 0 & ;\ \text{Otherwise}\end{cases}$

for p.d.f. If $f(x)$ is a p.d.f. then find F(1).

70. (a) Find the equations of those tangents to the circle $x^2 + y^2 = 52$ which are parallel to the straight line $2x + 3y = 6$

(OR)

(b) Solve the differential equation $(x+y)^2\frac{dy}{dx}=a^2$

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ANSWERS

PART - A

1. (b) of order 1 and degree 2
2. (a) $\left(\frac{-1}{2},-8\right)$
3. (b) $(-\pi, \pi]$
4. (d) $b:a$
5. (b) an asymptote parallel to y-axis
6. (c) ω^{n-k}
7. (d) $\vec{r}=\left(\vec{i}+3\vec{j}+5\vec{k}\right)+t\left(\vec{i}+5\vec{j}+\frac{3}{2}\vec{k}\right)$
8. (a) Gradient of the velocity / time graph
9. (d) 5
10. (c)(i), (iii), (iv)
11. (a) (6, 6)
12. (d) Two rows can have same number of zeros before the first non-zero entry
13. (b) $x = ce^{-my}$
14. (d) $\frac{1}{5\sqrt{2\pi}}$
15. (d) 60°
16. (c) 2
17. (b) $\frac{1}{3}$
18. (d) $\frac{1}{2}$
19. (d) $\left(25,\frac{1}{5}\right)$
20. (c) $f''(x_0) = 0$
21. (b) 2π
22. (a) $-t_2$
23. (a) 9
24. (c) $\frac{5\sqrt{3}}{2}$
25. (b) $\frac{2}{3}$
26. (a) 9
27. (a) the straight line $x=\frac{1}{4}$
28. (b) $f(x)\geq 0$
29. (b) $y = \pm\frac{5}{6}x$
30. (d) $\frac{1}{n}\cos^{n-1}x\sin x+\left(\frac{n-1}{n}\right)I_{n-2}$
31. (b) 0
32. (a) $\frac{dy}{dx}=\lambda y$
33. (c) $\frac{du}{dt}=\frac{\partial f}{\partial x}\cdot\frac{dx}{dt}+\frac{\partial f}{\partial y}\cdot\frac{dy}{dt}$
34. (b) $\begin{bmatrix}0&0\\0&5^{12}\end{bmatrix}$
35. (a) $[\vec{r}-\vec{a},\vec{u},\vec{v}]=0$
36. (b) $\cos x$
37. (a) k^3 det (A)
38. (a) (0, 0, –4)
39. (b) $\left(\log\frac{\Delta_2}{\Delta_1},\log\frac{\Delta_3}{\Delta_1}\right)$
40. (d) [2]

PART - B

41. ***Solution :***

$$\text{Let } A=\begin{bmatrix}0&1&2&1\\2&-3&0&-1\\1&1&-1&0\end{bmatrix}$$

$$\sim\begin{bmatrix}0&1&2&1\\2&-2&2&0\\1&1&-1&0\end{bmatrix}\quad R_2\to R_2+R_1$$

$$\sim\begin{bmatrix}0&1&2&1\\1&-1&1&0\\1&1&-1&0\end{bmatrix}\quad R_2\to\frac{1}{2}R_2$$

$$\sim\begin{bmatrix}0&1&2&1\\1&-1&1&0\\2&0&0&0\end{bmatrix}\quad R_3\to R_3+R_2$$

The last equivalent matrix is in echelon form. The number of non-zero rows in this matrix is three.

$\therefore\ \rho(A) = 3.$

42. ***Solution :***

Let $A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$. Then $|A| = \begin{vmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{vmatrix} = 2 \neq 0$

A is non-singular and hence A^{-1} exists

Cofactor of 3 = $A_{11} = \begin{vmatrix} -2 & 0 \\ 2 & -1 \end{vmatrix} = 2$

Cofactor of 1 = $A_{12} = -\begin{vmatrix} 2 & 0 \\ 1 & -1 \end{vmatrix} = 2$

Cofactor of –1 = $A_{13} = \begin{vmatrix} 2 & -2 \\ 1 & 2 \end{vmatrix} = 6$

Cofactor of 2 = $A_{21} = \begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} = -1$

Cofactor of –2 = $A_{22} = \begin{vmatrix} 3 & -1 \\ 1 & -1 \end{vmatrix} = -2$

Cofactor of 0 = $A_{23} = -\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} = -5$

Cofactor of 1 = $A_{31} = \begin{vmatrix} 1 & -1 \\ -2 & 0 \end{vmatrix} = -2$

Cofactor of 2 = $A_{32} = \begin{vmatrix} 3 & -1 \\ 2 & 0 \end{vmatrix} = -2$

Cofactor of –1 = $A_{33} = \begin{vmatrix} 3 & 1 \\ 2 & -2 \end{vmatrix} = -8$

$[A_{ij}] = \begin{bmatrix} 2 & 2 & 6 \\ -1 & -2 & -5 \\ -2 & -2 & -8 \end{bmatrix}$; adj A = $\begin{bmatrix} 2 & -1 & -2 \\ 2 & -2 & -2 \\ 6 & -5 & -8 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{2}\begin{bmatrix} 2 & -1 & -2 \\ 2 & -2 & -2 \\ 6 & -5 & -8 \end{bmatrix}$

$= \begin{bmatrix} 1 & -\frac{1}{2} & -1 \\ 1 & -1 & -1 \\ 3 & -\frac{5}{2} & -4 \end{bmatrix}$

43. ***Solution :***

The equation of the line passing through (1,1,–1) and (–1,0,1) is $\frac{x-1}{2} = \frac{y-1}{1} = \frac{z+1}{-2}$

It meets the xy-plane i.e. $z = 0$

$\therefore \frac{x-1}{2} = \frac{y-1}{1} = \frac{1}{-2} \Rightarrow x = 0, y = \frac{1}{2}$

The required point is $\left(0, \frac{1}{2}, 0\right)$.

44. ***Solution :***

(i) **Given**

$\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$

$(\vec{a} - \vec{d}) \times (\vec{b} - \vec{c})$

$= (\vec{a} \times \vec{b}) - (\vec{d} \times \vec{b}) - (\vec{a} \times \vec{c}) + (\vec{d} \times \vec{c})$

$= \vec{a} \times \vec{b} + \vec{b} \times \vec{d} - \vec{a} \times \vec{c} - \vec{c} \times \vec{d}$

$= \vec{a} \times \vec{b} + \vec{b} \times \vec{d} - \vec{b} \times \vec{d} - \vec{a} \times \vec{b}$

$(\because \vec{d} \times \vec{b} = -\vec{b} \times \vec{d})$

$= 0$ $\therefore (\vec{a} - \vec{d})$ and $(\vec{b} - \vec{c})$ are parallel.

(ii) $\overrightarrow{OA} = 2\vec{i} - 3\vec{j} + \vec{k}$; $\overrightarrow{OB} = 3\vec{i} + \vec{j} - 2\vec{k}$

$\overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB} = (2\vec{i} - 3\vec{j} + \vec{k}) - (3\vec{i} + \vec{j} - 2\vec{k})$

$= -\vec{i} - 4\vec{j} + 3\vec{k}$

$|\vec{r}| = \sqrt{(1)^2 + (-4)^2 + (3)^2}$

$= \sqrt{1 + 16 + 9} = \sqrt{26}$

$\therefore$ d.cs are $\vec{r} = \left(\frac{-1}{\sqrt{26}}, \frac{-4}{\sqrt{26}}, \frac{3}{\sqrt{26}}\right)$

45. ***Solution :***

Given $\alpha = -\sqrt{2} + i$ and $\beta = -\sqrt{2} - i$

$\therefore \alpha^2 + \beta^2 - \alpha\beta = (-\sqrt{2} + i)^2 + (-\sqrt{2} - i)^2 - (-\sqrt{2} + i)(-\sqrt{2} - i)$

$= 2 + i^2 - 2\sqrt{2}i + 2 + i^2 + 2\sqrt{2}i - (2 - i^2)$

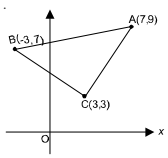
$= 2 - 1 + 2 - 1 - (2 + 1)$

$= 2 - 3$

$= -1$

46. ***Solution :***

Let A, B and C represent the complex numbers $7 + 9i$, $-3 + 7i$ and $3 + 3i$ in the Argand diagram respectively.



$AB = |(7 + 9i) - (-3 + 7i)|$

$= |10 + 2i| = \sqrt{10^2 + 2^2} = \sqrt{104}$

$BC = |(-3 + 7i) - (3 + 3i)|$

$= |-6 + 4i|$

$= \sqrt{(-6)^2 + 4^2} = \sqrt{36 + 16} = \sqrt{52}$

$CA = |(3 + 3i) - (7 + 9i)|$

$= |-4 - 6i|$

$= \sqrt{(-4)^2 + (-6)^2} = \sqrt{16 + 36} = \sqrt{52}$

$\Rightarrow AB^2 = BC^2 + CA^2$

$\Rightarrow \angle BCA = 90°$

Hence ΔABC is a right angled isosceles triangle.

47. ***Solution :***

Given displacement $x = 3\cos(2t - 4)$

Velocity $= \frac{dx}{dt} = -3\sin(2t - 4)(2) = -6\sin(2t - 4)$

Acceleration $= \frac{d^2x}{dt^2} = -6\cos(2t - 4)(2)$

$= -12\cos(2t - 4)$

Accleration at the end of $t = 2$ sec is $\left(\frac{d^2x}{dt^2}\right)_{\text{at } t=2}$

$= -12\cos(2(2) - 4)$

$= -12\cos(0) = -12(0) = 0$

Given, Kinetic energy K.E. $= \frac{1}{2} \times m \times v^2$

$= \frac{1}{2} \times (1) \times (0) = 0$

$[\because m$ is unit mass, $v = \frac{d^2x}{dt^2} = 0]$

48. ***Solution :***

(i) $f(x) = 4x^{3/5} - x^{8/5}$

$f'(x) = \frac{12}{5}x^{-2/5} - \frac{8}{5}x^{3/5}$

$= \frac{4}{5}x^{-2/5}(3 - 2x)$

Therefore $f'(x) = 0$ if $3 - 2x = 0$ i.e. if $x = \frac{3}{2}$. $f'(x)$ does not exist when $x = 0$. Thus the critical numbers are 0 and $\frac{3}{2}$.

(ii) $y = e^x$; $y'' = e^x > 0$ for x

Hence the curve is everywhere convex downward.

49. ***Solution :***

(i) Area of the Disc $A = \pi r^2$

$dA = 2\pi r\, dr$

Given $r = 24$ cm, $dr = 0.02$ cm

Approximate change in the area $\Delta A = 2\pi r\, dr$

$= 2\pi(24)(0.02)$

$= 0.96\pi$ cm^2

(ii) $A = \pi r^2$

Taking log on both sides

$\log A = \log \pi + 2 \log r$

Taking differential on both sides,

$\frac{1}{A} dA = 2 \cdot \frac{1}{r} dr$

Given $r = 24$ cm $dr = 0.02$ cm

The relative error in A i.e., $\frac{\Delta A}{A} = \frac{1}{A} dA = 2 \cdot \frac{1}{r} dr$

$= 2 \cdot \frac{1}{24} \times (0.02) = 0.0017$

The relative error in A is approximately 0.0017

50. ***Solution :***

The characteristic equation is $p^2 - 4p + 1 = 0$

$\Rightarrow p = \frac{4 \pm \sqrt{16 - 4}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$

C.F. $= A e^{(2+\sqrt{3})x} + B e^{(2-\sqrt{3})x}$

Let P.I. $= c_0 + c_1 x + c_2 x^2$

But P.I. is also a solution

$\therefore (D^2 - 4D + 1)(c_0 + c_1 x + c_2 x^2) = x^2$

i.e., $(2c_2 - 4c_1 + c_0) + (-8c_2 + c_1)x + c_2 x^2 = x^2$

$c_2 = 1$

$-8c_2 + c_1 = 0 \Rightarrow c_1 = 8$

$2c_2 - 4c_1 + c_0 = 0 \Rightarrow c_0 = 30$

P.I. $= x^2 + 8x + 30$.

Hence the general solution is y = C.F. + P.I.

$y = A e^{(2+\sqrt{3})x} + B e^{(2-\sqrt{3})x} + (x^2 + 8x + 30)$

51. ***Solution :***

p	q	$\sim q$	$p \vee (\sim q)$	$q \vee (p \vee (\sim q))$
T	T	F	T	T
T	F	T	T	T
F	T	F	F	T
F	F	T	T	T

The last column contains only T

$\therefore\ q \vee (p \vee (\sim q))$ is a tautology.

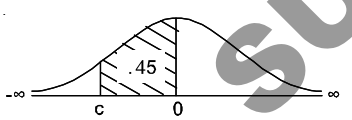
52. ***Solution :***

The compound statement $(p \wedge q) \vee (\sim r)$ consists of three simple statements p, q and r. Therefore, there must be $2^3 (= 8)$ rows in the truth table of $(p \wedge q) \vee (\sim r)$. The truth value of p remains at the same value of T or F for each of four consecutive assignments of logical values. The truth value of q remains at T or F for two assignments and that of r remains at T or F for one assignment.

p	q	r	$p \wedge q$	$\sim r$	$p \wedge q \vee (\sim r)$
T	T	T	T	F	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	F	T	T
F	T	T	F	F	F
F	T	F	F	T	T
F	F	T	F	F	F
F	F	F	F	T	T

53. ***Solution :***

(i)



$P(Z < c) = 0.05$ i.e., $P(-\infty < Z < c) = 0.05$

As area is < 0.5, c lies to the left of $Z = 0$.

From the area table Z value for the area 0.45 is 1.65. $\therefore c = -1.65$

(ii) Let the mean be $(m + 1)$ and the variance be m from the given data. [Since mean > variance in a binomial distribution]

$(m + 1)^2 - m^2 = 11 \Rightarrow m = 5$

$\therefore$ mean $= m + 1 = 6$

$\Rightarrow np = 6$; $npq = 5$

$\therefore q = \frac{5}{6}, p = \frac{1}{6} \quad \Rightarrow n = 36.$

54. ***Solution :***

Let X denote the number of success in throwing a die twice.

$\therefore$ X can take the values 0, 1 and 2

$\therefore$ P(Success) = P(S) = $\frac{3}{6} = \frac{1}{2}$

$\therefore$ P(Failure) = P(F) = $1 - \frac{1}{2} = \frac{1}{2}$

$P(X = 0) = P(FF) = P(F) . P(F) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

$P(X = 1) = P(SF \text{ or } FS) = P(SF) + P(FS)$

$= P(S) . P(F) + P(F) . P(S).$

$= 2\,P(F).\,P(S) = 2\left(\frac{1}{2} \times \frac{1}{2}\right) = \frac{1}{2}$

$P(X = 2) = P(SS) = P(S) . P(S) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

$\therefore$ Probability distribution function is

X	0	1	2
$P(X = x)$	$1/4$	$1/2$	$1/4$

$\text{Mean} = \Sigma[X] = \sum_{-\infty}^{\infty} x_i p_i = 0\left(\frac{1}{4}\right) + (1)\frac{1}{2} + (2)\frac{1}{4} = \frac{1}{2} + \frac{1}{2} = 1$

Variance = $E(X^2) - [E(X)]^2$

$E(X^2) = \sum_{-\infty}^{\infty} x^2_i\, p_i = (0^2)\left(\frac{1}{4}\right) + (1^2)\left(\frac{1}{2}\right) + (2^2)\left(\frac{1}{4}\right) = \frac{1}{2} + 1 = \frac{3}{2}$

$\therefore$ Variance (X) = $E(X^2) - [E(X)]^2 = \frac{3}{2} - (1)^2 = \frac{3}{2} - 1 = \frac{1}{2}$

$\therefore$ Mean = 1, Variance = $\frac{1}{2}$

55. (a) ***Solution :***

The transverse axis is parallel to x-axis and centre (h, k) is $(2, 5)$

$\therefore$ The equation of the hyperbola is $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

$\Rightarrow \frac{(y-5)^2}{a^2} - \frac{(x-2)^2}{b^2} = 1$

Given, distance between directrix, $\frac{2a}{e} = 15 \Rightarrow 2a = 15e$

and distance between foci,

$2ae = 20 \Rightarrow (15e)e = 20$

$\Rightarrow 15e^2 = 20 \Rightarrow = e^2 = \frac{20}{15}$

$\therefore \quad 2a = 15e \Rightarrow 4a^2 = 225\, e^2$

$\Rightarrow a^2 = \frac{225\, e^2}{4}$

$\Rightarrow a^2 = \frac{\overset{15}{\cancel{225}}}{\cancel{4}}\left(\frac{\overset{5}{\cancel{20}}}{\cancel{15}}\right) \Rightarrow a^2 = 75$

$$b^2 = a^2(e^2 - 1) = 75\left(\frac{20}{15} - 1\right) = 75\left(\frac{5}{75}\right) = 25$$

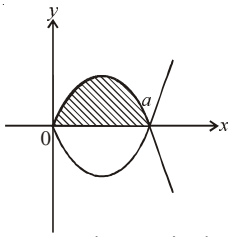
$\therefore$ The equation of the required hyperbola is

$$\frac{(y-5)^2}{75} - \frac{(x-2)^2}{25} = 1$$

(OR)

55. (b)

The curve is symmetric about x - axis and it passes through the origin. It cuts x - axis at $x = 0$, $x = a$ (twice). Clearly there is a loop between $x = 0$ and $x = a$.



As $x \to \infty$, $y \to \pm\infty$, The required volume is obtained by revolving the area bounded by the curve $2a^2 = x(x-a)^2$, $x = 0$, $x = a$ and x - axis, about x axis

$$\therefore \text{The required volume} = \pi\int_0^a y^2\,dx = \pi\int_0^a \frac{1}{2a}x(x-a)^2\,dx$$

$$= \frac{\pi}{2a}\int_0^a \left(x^3 - 2ax^2 + a^2x\right)dx$$

$$= \frac{\pi}{2a}\left[\frac{x^4}{4} - 2a\frac{x^3}{3} + a^2\frac{x^2}{2}\right]_0^a$$

$$= \frac{\pi}{2a}\left[\left(\frac{a^4}{4} - \frac{2a^4}{3} + \frac{a^4}{2}\right) - 0\right]$$

$$= \frac{\pi}{2a}\left[\left(\frac{3a^4 - 8a^4 + 6a^4}{12}\right)\right]$$

$$= \frac{\pi a^3}{24} \text{ cubic units.}$$

PART - C

56. ***Solution :***

$$\Delta = \begin{vmatrix} 1 & 1 & 2 \\ 2 & 2 & 4 \\ 3 & 3 & 6 \end{vmatrix} = 0 \qquad \Delta_x = \begin{vmatrix} 4 & 1 & 2 \\ 8 & 2 & 4 \\ 12 & 3 & 6 \end{vmatrix} = 0$$

$$\Delta_y = \begin{vmatrix} 1 & 4 & 2 \\ 2 & 8 & 4 \\ 3 & 12 & 6 \end{vmatrix} = 0, \qquad \Delta_z = \begin{vmatrix} 1 & 1 & 4 \\ 2 & 2 & 8 \\ 3 & 3 & 12 \end{vmatrix} = 0$$

Since $\Delta = 0$ and $\Delta_x = \Delta_y = \Delta_z = 0$ also all 2×2 minors of Δ, Δ_x, Δ_y and Δ_z are zero, by case 2(c), it is consistent and has infinitely many solutions.
($\because$ all 2×2 minors zero and atleast one of a_{ij} of $\Delta \neq 0$, the system is reduced to single equation).
Let us take $x = s$ and $y = t$, we get from equation (1)

$z = \frac{1}{2}(4 - s - t)$ $\therefore$ the solution set is

$$(x, y, z) = \left(s, t, \frac{4-s-t}{2}\right), s, t \in \mathbb{R}$$

Particular numerical solution for

$(x, y, z) = (1, 1, 1)$ when $s = t = 1$

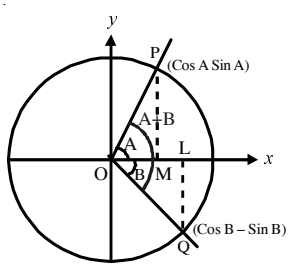
$(x, y, z) = \left(-1, 2, \frac{3}{2}\right)$ when $s = -1$, $t = 2$

57. ***Solution :***

Take the points P and Q on the unit circle with centre at the origin O.
Assume that OP and OQ makes angles A and B with x – axis respectively.

$$\therefore \angle POQ = \angle POx + \angle QOx = A + B$$

Clearly the coordinates of P and Q are (cos A, Sin A) and (Cos B, – Sin B) respectively.
Take the unit vector $\vec{i}$ and $\vec{j}$ along x and y axes.
Draw PM $\perp$ x axis and QL $\perp$ x axis



$\vec{OP} = \vec{OM} + \vec{MP} = \cos A\,\vec{i} + \sin A\,\vec{i}$
$\vec{OQ} = \vec{OL} + \vec{LQ} = \cos B\,\vec{i} - \sin B\,\vec{i}$
By value,
$\vec{OP} \cdot \vec{OQ} = (\cos A\,\vec{i} + \sin A\,\vec{j}) \cdot (\cos B\,\vec{i} - \sin B\,\vec{j})$ ..... (1)

$= \cos A \cos B - \sin A \sin B$

By definition, $\vec{OP} \cdot \vec{OQ} = |\vec{OP}|\,|\vec{OQ}|\,(\cos A + B)$
$= (1)(1)(\cos A + B)$
$= \cos(A + B)$ ..... (2)

From (1) and (2) we get,

$\cos(A + B) = \cos A \cos B - \sin A \sin B$

58. ***Solution :***

Given $(x_1, y_1, z_1) = (3, 4, 2)$; $(x_2, y_2, z_2) = (2, -2, -1)$; $(x_3, y_3, z_3) = (7, 0, 1)$

Vector Equation:

The vector equation of the required plane is

$\vec{r} = (1 - s - t)\,\vec{a} + s\,\vec{b} + t\,\vec{c}$

$\vec{r} = (1 - s - t)\,(3\vec{i} + 4\vec{j} + 2\vec{k}) + s\,(2\vec{i} - 2\vec{j} - \vec{k}) + t\,(7\vec{i} + \vec{k})$

$\vec{r} = (3\vec{i} + 4\vec{j} + 2\vec{k}) + s\,(-\vec{i} + 6\vec{j} - 3\vec{k}) + t\,(4\vec{i} - 4\vec{j} - \vec{k})$

Cartesian Equation :

The cartesian equation of the required form is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x - 3 & y - 4 & z - 2 \\ -1 & -6 & -3 \\ 4 & -4 & -1 \end{vmatrix} = 0$$

$\Rightarrow (x - 3)(6 - 12) - (y - 4)(1 + 12) + (z - 2)(4 + 24) = 0$

$\Rightarrow -6x + 18 - 13y + 52 + 28z - 56 = 0$

$\Rightarrow -6x - 13y + 28z + 14 = 0$

$\Rightarrow 6x + 13y - 28z - 14 = 0$

59. ***Solution :***

$x^4 - x^3 + x^2 - x + 1 = 0 \Rightarrow 1 - x + x^2 - x^3 + x^4 = 0$

Now the given series is an G.P with

$a = 1, r = -x, n = 5$

$$\left[\therefore 1 + r + r^2 + \ldots\ldots\ldots + r^n = \frac{a(r^n - 1)}{r - 1} \text{ where } r \neq -1\right]$$

$$\Rightarrow \frac{x^5 + 1}{x + 1} = 0, \text{ Where } x \neq -1$$

Hence $x^5 = -1 = \cos\pi + i\sin\pi$

$\therefore x = [\cos(2k\pi + \pi)\ i\sin(2k\pi + \pi)]^{1/5}$

$= \cos\frac{2k\pi + \pi}{5} + i\sin\frac{2k\pi + \pi}{5}$, Where $k = 0, 1, 3, 4$

$= \cos\frac{\pi}{5}(2k + 1) + i\sin\frac{\pi}{5}(2k + 1)$, Where $k = 0, 1, 3, 4$

The values are $\cos\frac{\pi}{5} + i\sin\frac{\pi}{5}$; $\cos\frac{3\pi}{5} + i\sin\frac{3\pi}{5}$;

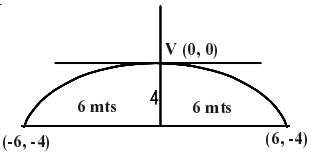
$\cos\pi + i\sin\pi$; $\cos\frac{7\pi}{5} + i\sin\frac{7\pi}{5}$; $\cos\frac{9\pi}{5} + i\sin\frac{9\pi}{5}$

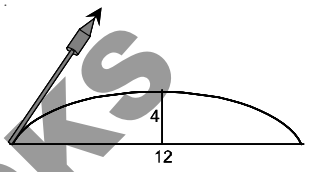
But $\cos\pi + i\sin\pi = 1$

$\therefore$ The roots are $\cos\frac{\pi}{5} + i\sin\frac{\pi}{5}$, $\cos\frac{3\pi}{5} + i\sin\frac{3\pi}{5}$,

$\cos\frac{7\pi}{5} + i\sin\frac{7\pi}{5}$, $\cos\frac{9\pi}{5} + i\sin\frac{9\pi}{5}$

60. ***Solution :***





The equation of the parabola is of the form $x^2 = -4ay$ (by taking the vertex at the origin). It passes through $(6, -4)$

$\therefore 36 = 16a \Rightarrow a = \frac{9}{4}$

The equation is $x^2 = -9y$ ...(1)

Find the slope at $(-6, -4)$

Differentiating (1) with respect to x, we get

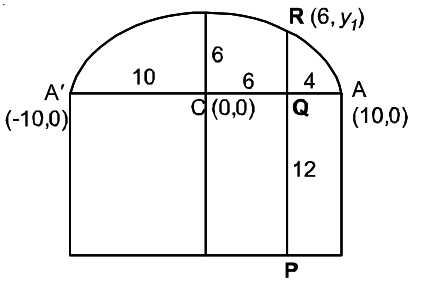
$2x = -9\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = -\frac{2}{9}x$

At $(-6, -4)$, $\frac{dy}{dx} = -\frac{2}{9} \times -6 = \frac{4}{3}$ i.e. $\tan\theta = \frac{4}{3}$

$$\theta = \tan^{-1}\left(\frac{4}{3}\right)$$

$\therefore$ The angle of projections is $\tan^{-1}\left(\frac{4}{3}\right)$

61. ***Solution :***



Let PQR be the height of the ceiling which is 4 feet from the wall.

From the diagram PQ = 12 ft. To find the height QR.

Since the width is 20ft, take A, A′ as vertices with A as (10, 0) and A′ as (–10, 0). Take the midpoint of AA′ as the centre which is (0, 0).

From the diagram AA′ $= 2a = 20 \Rightarrow a = 10$

and $b = 18 - 12 = 6$

$$\therefore \frac{x^2}{100} + \frac{y^2}{36} = 1$$

Let QR be y_1 then R is $(6, y_1)$

Since R lies on the ellipse, $\frac{36}{100} + \frac{y_1^2}{36} = 1 \Rightarrow y_1 = 4.8$

PQ + QR = 12 + 4.8

∴ The required height of the ceiling is 16.8 feet.

62. ***Solution :***

One asymptote is $x + 2y - 5 = 0$

∴ The other asymptote is the line perpendicular to the given asymptote

i.e., $2x - y + k = 0$

∴ Equation of the Rectangular hyperbola is

$(x + 2y - 5)(2x - y + k) + c = 0$

This passes through (6, 0)

$\therefore [6 + 2(0) - 5][2(6) - 0 + k] + c = 0$

$(1)(12 + k) + c = 0$

$k + c = -12$ ..... (1)

This passes through (–3, 0)

$[-3 + 2(0) - 5][2(-3) - 0 + k] + c = 0$

$(-8)(-6 + k) + c = 0$

$48 - 8k + c = 0$

$8k - c = 48$ ..... (2)

Solving (1) & (2)

$$\begin{aligned} k + c &= -12 \\ 8k - c &= 48 \\ \hline 9k &= 36 \Rightarrow k = \frac{36}{9} = 4 \end{aligned}$$

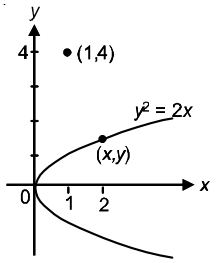
Subst. $k = 4$ in (1) $\Rightarrow 4 + c = -12$

$\therefore c = -12 - 4 = -16$

∴ Equation of the rectangular hyperbola is

$(x + 2y - 5)(2x - 7 + 4) - 16 = 0.$

63. ***Solution :***



Let (x,y) be the point on the parabola $y^2 = 2x$. The distance between the points (1,4) and

(x,y) is $d = \sqrt{(x-1)^2 + (y-4)^2}$.

(x,y) lies on $y^2 = 2x \Rightarrow x = \frac{y^2}{2}$,

so $d^2 = f(y) = (\frac{y^2}{2} - 1)^2 + (y - 4)^2$

(Note that the minimum of d occurs at the same point as the minimum of d^2)

Now $f'(y) = 2\left(\frac{y^2}{2} - 1\right)(y) + 2(y - 4)$

$= y^3 - 8 = 0$ at a critical point.

$y^3 - 8 = 0 \Rightarrow y = 2$ (since $y^2 + 2y + 4 = 0$ is not possible)

Observe that $f'(y) < 0$ when $y < 2$ and $f'(y) > 0$ when $y > 2$, so by the first derivate test, for absolute extrema, the absolute minimum occurs when $y = 2$.

The corresponding value of x is $x = \frac{y^2}{2} = 2$.

Thus the point on $y^2 = 2x$ closest to (1,4) is (2,2).

64. ***Solution :***

$$u = \frac{x}{y^2} - \frac{y}{x^2}$$

$$\frac{\partial u}{\partial x} = \frac{1}{y^2}.1 - y\left(-\frac{2}{x^3}\right) = \frac{1}{y^2} + \frac{2y}{x^3};$$

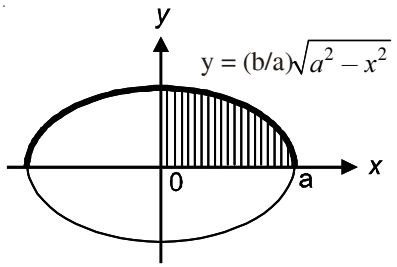
$$\frac{\partial u}{\partial y} = x.\left(\frac{-2}{y^3}\right) - \frac{1}{x^2} = -\frac{2x}{y^3} - \frac{1}{x^2};$$

$$\frac{\partial^2 u}{\partial x\,\partial y} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial x}\left(-\frac{2x}{y^3} - \frac{1}{x^2}\right) = -\frac{2}{y^3} + \frac{2}{x^3} \quad ..... (1)$$

$$\frac{\partial^2 u}{\partial y\,\partial x} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right) = \frac{\partial}{\partial y}\left(\frac{1}{y^2} + \frac{2y}{x^3}\right) = -\frac{2}{y^3} + \frac{2}{x^3} \quad ..... (2)$$

From (1) and (2) $\frac{\partial^2 u}{\partial x\,\partial y} = \frac{\partial^2 u}{\partial y\,\partial x}$

65. ***Solution :***



The curve is symmetric about both axes.
∴ Area of the ellipse = 4 × Area of the ellipse in the I quadrant.

$$I = 4\int_0^a y\,dx$$

$$= 4\int_0^a \frac{b}{a}\sqrt{a^2 - x^2}\;dx$$

$$= \frac{4b}{a}\int_0^a \sqrt{a^2 - x^2}\;dx = \frac{4b}{a}\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a$$

$$= \frac{4b}{a}\left[0 + \frac{a^2}{2}\sin^{-1}(1) - 0\right] = \frac{4b}{a}\left(\frac{a^2}{2}\left(\frac{\pi}{2}\right)\right)$$

$= \pi\, ab$ sq. units.

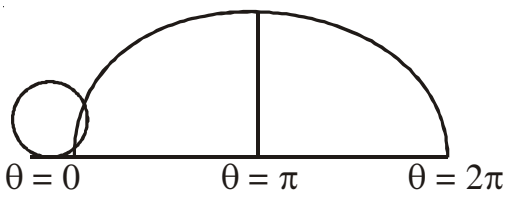
By using parametric form i.e.,

$$4\int_0^a y\,dx = 4\int_0^{\pi/2} b\sin\theta(-a\sin\theta)\,d\theta,$$

we get the same area.

66. ***Solution :***

The period of one arc is 2π i.e., 0 to 2π
∴ $t = 0$ to $t = \pi$ gives only length of the half arc.



$$x = a\,(t - \sin t) \Rightarrow \frac{dx}{dt} = a\,(1 - \cos t)$$

$$y = a\,(1 - \cos t) \Rightarrow \frac{dy}{dt} = a\sin t$$

$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = a^2\,[(1 - \cos t)^2 + \sin^2 t]$$

$$= 2a^2\,(1 - \cos t)$$

$$= 2a^2 . 2\sin^2\frac{t}{2} = 4a^2\sin^2\left(\frac{t}{2}\right)$$

$$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = 2a\sin\frac{t}{2}$$

The required length $= \int_0^{\pi}\sqrt{\left(\frac{dx}{dt}\right) + \left(\frac{dy}{dt}\right)^2}\,dt = \int_0^{\pi} 2a\sin\frac{t}{2}\,dt$

$$= 2a.\left[-\frac{\cos\left(\frac{t}{2}\right)}{1/2}\right]_0^{\pi} = -4a\left[\cos\frac{\pi}{2} - \cos\theta\right] = 4a\,[0 - 1]$$

$= 4\,a$ units.

Note : This curve is known as Cycloid.

67. ***Solution :***

Let T be the temperature of the coffee at any time and S be the room temperature.

Then the required differential equation is $\frac{dT}{dt} \propto T - S \Rightarrow \frac{dT}{dt} = k\,(T - S)$ where k is a constant.

$$\Rightarrow \frac{dT}{T\text{-}S} = k\,dt$$

Integrating we have,

$\log\,(T - S) = kt + \log c \Rightarrow T - S = ce^{kt}$ ..... (1)

When T = 100°, S = 15°, $t = 0 \Rightarrow (1) \Rightarrow 100° - 15°$

$= ce^0 \Rightarrow c = 85°$

$\therefore\ T - S = 85°\,e^{kt}$

Given that the coffee cools to 60° in 5 minutes

i,e., when $t = 5$, T = 60°

$(2) \Rightarrow 60° - 15° = 85e^{5k} \Rightarrow e^{5k} = \frac{45°}{85°}$

$\therefore$ After further interval of 5 minutes (i.e., $t = 10$) the temperature is $T - S = 85e^{10k}$

$$\Rightarrow T - 15° = 85° \left(\frac{45}{85}\right)^2 \Rightarrow T = 15 + \frac{45 \times 45}{85} = 38.82° \text{ C}$$

68. ***Solution :***

Let $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$; $A = \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}$, $B = \begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix}$;

$I = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$; $D = \begin{pmatrix} 0 & \omega^2 \\ \omega & 0 \end{pmatrix}$; $E = \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix}$

Let G = {I, A, B, C, D, E}

$$I * I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} * \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

$$I * A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} * \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix} = \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix} = A$$

$$I * B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} * \begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix} = \begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix} = B$$

$$I * C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} * \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = C$$

$$I * D = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} * \begin{pmatrix} 0 & \omega^2 \\ \omega & 0 \end{pmatrix} = \begin{pmatrix} 0 & \omega^2 \\ \omega & 0 \end{pmatrix} = D$$

$$I * E = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} * \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix} = E$$

$$A * I = \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix} * \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix} = A$$

$$A * A = \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix} * \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix} = \begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix} = B$$

$$A * B = \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix} * \begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

$$A * C = \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix} * \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix} = E$$

$$A * D = \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix} * \begin{pmatrix} 0 & \omega^2 \\ \omega & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = C$$

$$A * E = \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix} * \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & \omega^2 \\ \omega & 0 \end{pmatrix} = D$$

$$B * I = \begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix} * \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix} = B$$

$$B * A = \begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix} * \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

$$B * B = \begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix} * \begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix} = \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix} = A$$

$$B * C = \begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix} * \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & \omega^2 \\ \omega & 0 \end{pmatrix} = D$$

$$B * D = \begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix} * \begin{pmatrix} 0 & \omega^2 \\ \omega & 0 \end{pmatrix} = \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix} = E$$

$$B * E = \begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix} * \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = C$$

$$C * I = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} * \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = C$$

$$C * A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} * \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix} = \begin{pmatrix} 0 & \omega^2 \\ \omega & 0 \end{pmatrix} = D$$

$$C * B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} * \begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix} = \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix} = E$$

$$C * C = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} * \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = I$$

$$C * D = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} * \begin{pmatrix} 0 & \omega^2 \\ \omega & 0 \end{pmatrix} = \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix} = A$$

$$C * E = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} * \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix} = \begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix} = B$$

$$D * I = \begin{pmatrix} 0 & \omega^2 \\ \omega & 0 \end{pmatrix} * \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & \omega^2 \\ \omega & 0 \end{pmatrix} = D$$

$$D * A = \begin{pmatrix} 0 & \omega^2 \\ \omega & 0 \end{pmatrix} * \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix} = \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix} = E$$

$$D * B = \begin{pmatrix} 0 & \omega^2 \\ \omega & 0 \end{pmatrix} * \begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = C$$

$$D * C = \begin{pmatrix} 0 & \omega^2 \\ \omega & 0 \end{pmatrix} * \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix} = B$$

$$D * D = \begin{pmatrix} 0 & \omega^2 \\ \omega & 0 \end{pmatrix} * \begin{pmatrix} 0 & \omega^2 \\ \omega & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

$$D * E = \begin{pmatrix} 0 & \omega^2 \\ \omega & 0 \end{pmatrix} * \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix} = \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix} = A$$

$$E * I = \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix} * \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix} = E$$

$$E * A = \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix} * \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = C$$

$$E * B = \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix} * \begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix} = \begin{pmatrix} 0 & \omega^2 \\ \omega & 0 \end{pmatrix} = D$$

$$E * C = \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix} * \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix} = A$$

$$E * D = \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix} * \begin{pmatrix} 0 & \omega^2 \\ \omega & 0 \end{pmatrix} = \begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix} = B$$

$$E * E = \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix} * \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

Cayley's table

·	I	A	B	C	D	E
I	I	A	B	C	D	E
A	A	B	I	E	C	D
B	B	I	A	D	E	C
C	C	D	E	I	A	B
D	D	E	C	B	I	A
E	E	C	D	A	B	I

(i) Closure axiom :

From the table it is clear that G is closed under '*'

∴ Closure property is true.

(ii) Associative axiom :

Matrix multiplication is always associative.

(iii) Identity axiom :

From the table it is clear that I is the identity element.

(iv) Inverse axiom :

$I * I = I \Rightarrow$ Inverse of I is I

$A * B = I \Rightarrow$ Inverse of A is B

$B * A = I \Rightarrow$ Inverse of B is A

$C * C = I \Rightarrow$ Inverse of C is C

$D * D = I \Rightarrow$ Inverse of D is D

$E * E = E \Rightarrow$ Inverse of E is E

∴ (G, *) is a group with required matrix multiplication.

69. ***Solution :***

To verify $f(x)$ is a p.d.f., we have to

P.I. $\int_{-\infty}^{\infty} f(x)\,dx = 1$

x	0	∞
t	0	∞

Since $x > 0$, $\int_0^{\infty} 30x^4 e^{-6x^5}\,dx = 30\int_0^{\infty} x^4 e^{-6x^5}\,dx$

Put $x^5 = t \Rightarrow 5x^4\,dx = dt \Rightarrow x^4\,dx = \frac{dt}{5}$

$\therefore\ 30\int_0^{\infty} e^{-6t}\cdot\frac{dt}{5} = \frac{30}{5}\int_0^{\infty} e^{-6t}dt = 6\left[\frac{e^{-6t}}{-6}\right]_0^{\infty} = -\left[e^{-6t}\right]_0^{\infty}$

$= -[e^{-\infty} - e^0] = -[0-1] = 1$

$\therefore f(x)$ is a p.d.f.

Put $x^5 = t$
$\Rightarrow x^4\,dx = \frac{dt}{5}$

Now, $F(1) = P(X \le 1) = \int_0^1 f(x)\,dx = \int_0^1 30x^4 e^{-6x^5}\,dx$

$= 30\int_0^1 x^4 e^{-6x^5}\,dx = \frac{30}{5}\int_0^1 e^{-6t}\,dt = 6\left[\frac{e^{-6t}}{6}\right]_0^1$

$= -\left[e^{-6t}\right]_0^1 = -[e^{-6} - e^0] = -[e^{-6} - 1] = 1 - e^{-6}$

$\therefore F(1) = 1 - e^{-6}$

70. (a) ***Solution :***

Let (x_1, y_1) be the point on the curve $x^2 + y^2 = 52$ at which the tangent is parallel to the line $2x + 3y = 6$.

Slope of the tangent at (x_1, y_1) = Slope of the line $2x + 3y = 6$. ..... (1)

$x^2 + y^2 = 52 \Rightarrow 2x + 2y\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{-2x}{2y} = \frac{-x}{y}$

$\therefore$ Slope of $x^2 + y^2 = 52$ at (x_1, y_1) is $\left(\frac{dy}{dx}\right)_{\text{at }(x_1, y_1)} = -\frac{-x_1}{y_1}$

Slope of the line $2x + 3y = 6$

$2x + 3y = 6 \Rightarrow 3y = -2x + 6$

$\Rightarrow y = \frac{-2}{3}x + 2$

Slope of $2x + 3y = 6 = \frac{-2}{3}$

(x_1, y_1) lies on the circle $x^2 + y^2 = 52$

$x_1^2 + y_1^2 = 52 \Rightarrow \left(\frac{2}{3}y_1\right)^2 + y_1^2 = 52$

$\frac{4}{9}y_1^2 + y_1^2 = 52 \Rightarrow 13y_1^2 = 52 \times 9$

$y_1^2 = \frac{52 \times 9}{13} = 36 \Rightarrow y_1 = \pm 6$

When $y_1 = 6$ $x_1 = \frac{2}{3}(6) = 4$;

When $y_1 = -6$ $x_1 = \frac{2}{3}x - 6 = -4$

$\therefore$ The points are (4, 6) and (–4, –6)

Equation of the tangent at (4, 6) and having slope $-\frac{2}{3}$ is

$y - y_1 = m(x - x_1)$

$y - 6 = -\frac{2}{3}(x - 4)$

$3y - 18 = -2x + 8$

$2x + 3y - 26 = 0$

Equation of the tangent at (– 4, – 6) and having slope $\frac{-2}{3}$ is

$(y + 6) = \frac{-2}{3}(x + 4)$

$\Rightarrow 3y + 18 + 2x + 8 = 0$

$\Rightarrow 2x + 3y + 26 = 0$

(OR)

(b) ***Solution :***

Put $x + y = z$. Differentiating with respect to x we get

$$1+\frac{dy}{dx}=\frac{dz}{dx} \text{ i.e., } \frac{dy}{dx}=\frac{dz}{dx}-1$$

The given equation becomes $z^2\left(\frac{dz}{dx}-1\right)=a^2$

$$\Rightarrow \frac{dz}{dx}-1=\frac{a^2}{z^2} \text{ (or) } \frac{z^2}{z^2+a^2}dz=dx$$

Integrating we have, $\int \frac{z^2}{z^2+a^2}dz=\int dx$

$$\int \frac{z^2+a^2-a^2}{z^2+a^2}dz=x+c \Rightarrow \int\left(1-\frac{a^2}{z^2+a^2}\right)dz=x+c$$

$$\Rightarrow z-a^2.\frac{1}{a}\tan^{-1}\frac{z}{a}=x+c$$

$$\Rightarrow x+y-a\tan^{-1}\left(\frac{x+y}{a}\right)=x+c \quad (\because z=x+y)$$

i.e., $y-a\tan^{-1}\left(\frac{x+y}{a}\right)=c$. which is the required solution.